Credit Derivatives

Master gestion des risques

Finance Factory

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Definition

Credit risk is the oldest form of financial risk (ever since lending began)

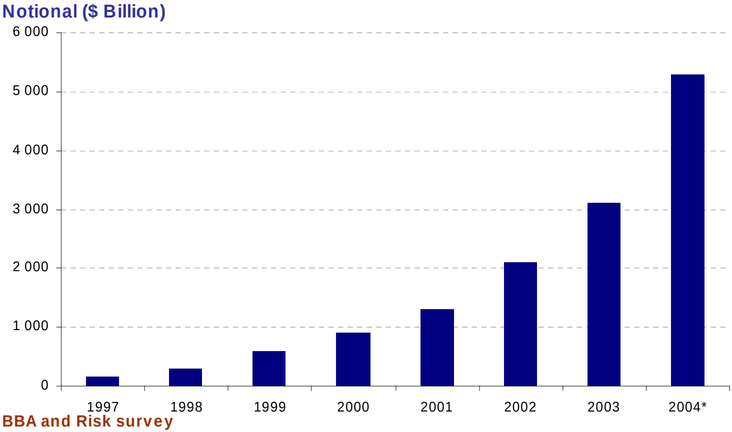
Definition (Credit derivatives)

Credit Derivatives are synthetic contracts to buy or sell protection against credit-related losses (either on a single name basis (default swap) or on a portfolio basis (default basket)).

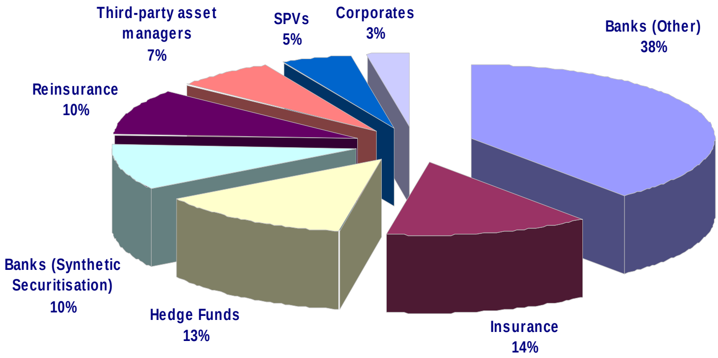
Definition (2/2)

- Credit market is more efficient : credit risk can now be shorted in a cost effective way.
- More efficient risk allocation is possible.
- Synthetic exposure can be structured independently of the maturity profile of a given reference credit's debt.
- Credit risk can now be separated from other factors like interest rate risk or funding costs, and even from spread risk (spread options).

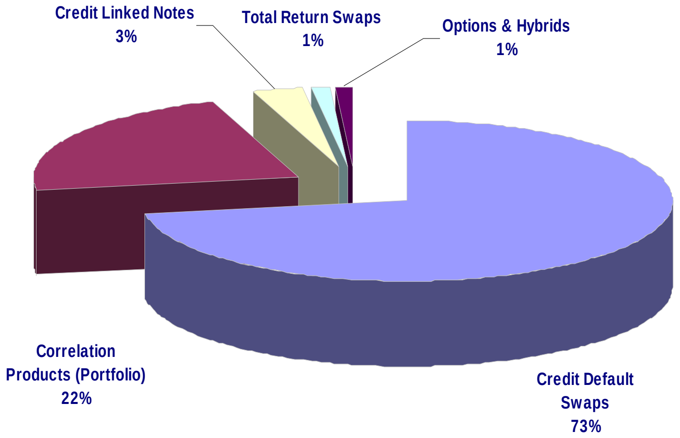
Credit Derivatives Market



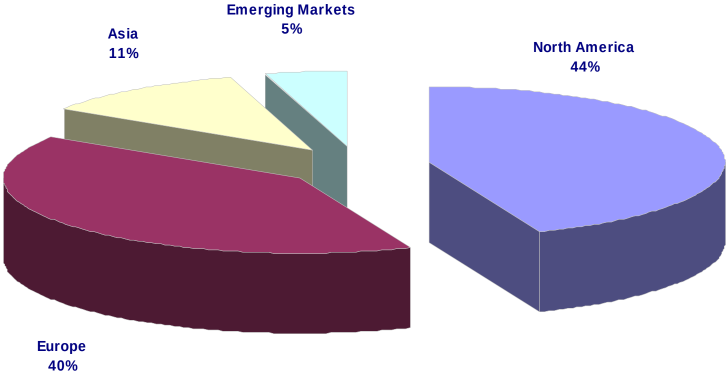
End-User Breakdown



Products Breakdown



Default Swap Market



Who are investors ?

- *Equity* : hedge funds, prop trading desks, investment books, ... that are not marked to market.
- *Mezzanine* : fund managers, banks, ... that need investment grade ratings.
- *Senior* : re-insurers, ... that need relatively low spread in return for assuming tail risk.

How are tranches different from baskets ?

- Tranches reference much larger portfolios than basket. Consequently, they reduce the idiosyncratic risk and so improve diversification.
- Credit monitoring is more demanding for CDO tranches than for basket.
- Tranches are more flexible because both subordination and tranche width can be selected.
- Tranches behave in a less digital way than baskets : default baskets are triggered by a single default, tranches are gradually eaten up, once the subordination level has been breached.

Where do we come from ?

In the last two years, many dealers have been trading mezzanine tranches with investors.

Now, they have to deal with the corresponding imbalances in their correlation books. This has widened equity spreads relative to mezzanine tranches.

Where do we go ?

- Standardized portfolios : TRACX, CDX, TREMONT Index.
- CDO square tranches.
- Equity Default Swap.
- CFO : Collateralized Fund Obligation (or Hedge fund CDO).
- Traders imagination is boundless !

Definition

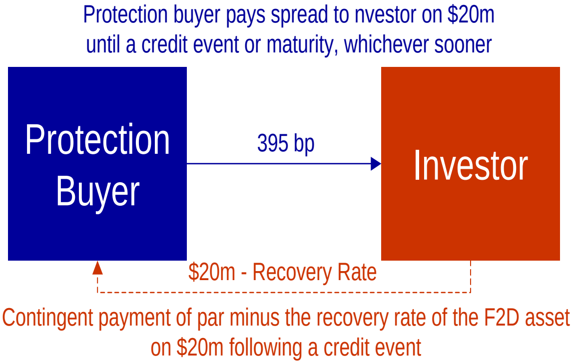
Definition

The protection buyer (synthetically short the credit) pays a premium of x basis points per annum on a notional amount to the seller (synthetically long the credit) in exchange for taking the risks on a credit event of any of the reference credits.

In this instance this reference credit will be determined to be the first to default of the pool of reference credits.

In the event that a reference credit defaults, the buyer has the right to deliver to the seller defaulted obligations with claims equal to the notional amount of the contract, with the recovery amount being determined through market quotations.

Understanding the Mechanics



Reference Basket

Name	Spread
A	36 bp
B	103 bp
C	51 bp
D	36 bp
E	64 bp
F	185 bp
Total	**474 bp**

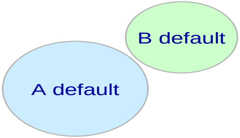
Why do a default basket trade ?

- Investor : Leveraging risk of a given portfolio.
  Maximum downside is the same as for a single default swap,
  Trigger probability is higher.
- Hedger : Cheapens the cost of protection.
  Implements the view that at most one or two credits out of basket will default.

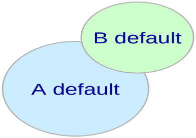
Remark : F2D spread often quoted as a percentage of the sum of spreads. Investors can expect to receive between 70% and 85%.

Default Baskets are Correlation Products !

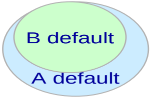
Minimal default dependance:
F2D protection is worth the same as protection on both credits.
S2D protection is worthless.



Positive default dependance:
F2D protection is worth more than the maximum protection, but less than the sum.
S2D protection has some value, but less than the minimum.



Maximal default dependance:
F2D protection is worth the same as the maximum protection.
S2D protection is worth the same as the minimum protection.

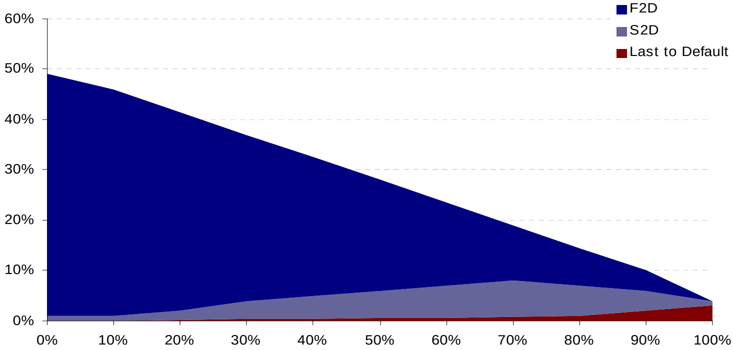


Conclusion with two products

The two credits analysis is somewhat special but we can conclude that :

- F2D : The value of the protection is greater than the maximum of the value of protection on each individual credit, but small than the sum of the individual protection values. F2D protection decreases in value as default dependance increases. A buyer of F2D protection is "short correlation".
- S2D : The value of the protection can be practically worthless, but approaches the minimum of the individual default protection in the two credit case.

Correlation Products



Correlation trading

Definition

Correlation trading is the art of dynamically managing a book of liabilities, and so monitoring correlation and model risk.

- A market for implied correlation is evolving (an on-going process for the moment).
- Not a consensus for the model (like B&S). Nevertheless, the Gaussian one-factor model is becoming the benchmark reference model.

Correlation book

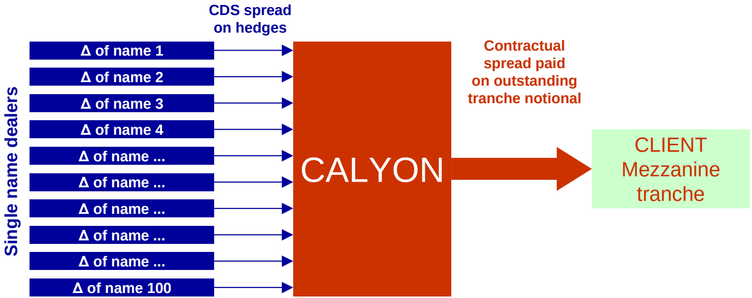
The typical correlation book is composed of a set of

- CDO tranches
- N^{th} to default baskets
- other credit products as CDS, bonds, loans, ...

and like an options book (correlation replace volatility), every tranche has a number of greeks (delta, gamma, ValueOnDefault, correlation sensitivity, ...) that have to be hedged by the trading team.

Hedging

Dealers hedges Spread Risk by selling CDS protection. The first order risk is changes in spreads for names in portfolio.



Hazard rate

Let denote by τ the time of default of an obligor. In many cases, we can write the survival probabilities as

$$\mathbf{P}[\tau > T|F_t] = exp\left(-\int_t^T h\left(t,s\right)ds\right)$$

The function $h(t,s)$ is called the *hazard rate* of default.

- $h(t,s)$ must be non negative
- $h(t,s)$ may be stochastic (as a function of time t)
- If we note $p(t,T)$ the density of τ

$$h\left(t,s\right) = \frac{p(t,T)}{1 - F(t,T)}$$

Intensities

The default indicator $N'(t) = 1_{\tau \leq t}$ is a non decreasing, adapted, integrable stochastic process. Therefore, it has a unique Doob-Meyer decomposition

$$M(t) = N'(t) - A(t)$$

where $M(t)$ is a martingale and $A(t)$ a predictable process of finite variation (predictable compensator).
If A can be represented as an integral

$$A(T) = \int_t^T \lambda(t)dt$$

$\lambda(t)$ is called the *intensity* of τ.

Important remarks

A predictable time of default τ does not have an intensity.
If τ is predictable, so is $N'(t)$. Then $A(t) = N'(t)$ and we can not write $A(t)$ as an integral.

Classical firm's value models do not have intensities. This can be proved formally, but it is natural to conjecture

$$h(t,t) = \lambda(t).$$

Exponentially distributed times

$$P(0,T) = \mathbf{P}[\tau > T] = e^{-\eta T}$$

An exponential time can also be viewed as the time of the first jump of a Poisson process with intensity η

$$\begin{aligned} h(t,T) &= \eta \\ \lambda(t) &= \eta 1_{\tau > t} \\ A(t) &= \eta(t \bigwedge \tau) \end{aligned}$$

Constant Time

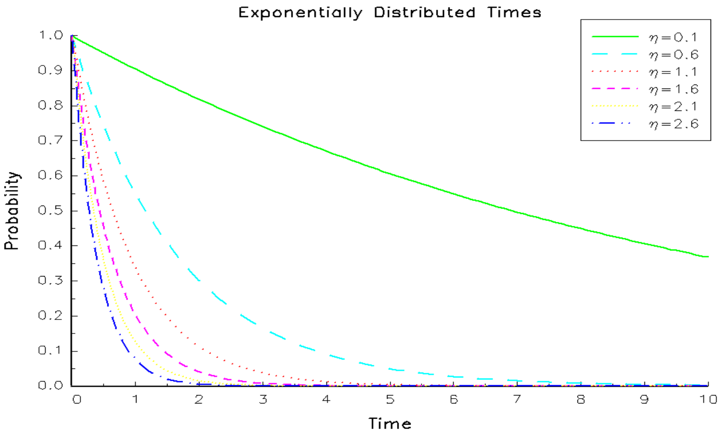
The time of default τ is constant and so the intensity $\lambda(t)$ does not exist.

Inhomogeneous Poisson Process : IPP

The increments of an inhomogeneous Poisson process $N(t)$ with intensity function $\lambda(t)$ satisfy

$$\mathbf{P}[N(T) - N(t) = n] = \frac{1}{n!}\left(\int_t^T \lambda(s)ds\right)^n exp\left(-\int_t^t \lambda(s)ds\right)$$

Example


Exponentially Distributed Times
η=0.1
η=0.6
η=1.1
η=1.6
η=2.1
η=2.6
Probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
Time

Introduction

- Approche structurelle basée sur le pricing d'options (B&S - Merton)
- Vision optionnelle de la dette
- 1^{er} exemple par Merton sur un zéro coupon. La formule fermée explicite l'impact :
  - du taux sans risque
  - de la volatilité du MtM des actifs
  - du levier d (ratio dette sur valeur des actifs)

Hypothèses

Hypothèses classiques sur la valorisation de la dette :

- Absence d'opportunité d'arbitrage (existence de prix)
- Les marchés sont complets (unicité du prix)
  (réplication de tout produit grace à une stratégie d'arbitrage)
- Le cours f représente les anticipations de valorisation de la firme à T

Dynamique

Évolution de la valeur des actifs suivant la dynamique :

$$\mathrm{d}V(t) = (\alpha V(t) - C)\,\mathrm{d}t + \sigma V(t)\,\mathrm{d}W(t)$$

où :

- α : taux de rendement instantané des actifs de la firme
- C : montant des payouts par unité de temps
- σ^2 : variance du processus de rendement

Conditions d'émission de la dette

- Paiement d'un montant B bullet à la maturité T
- Aucune émission nouvelle avant la maturité T

$$C = 0$$

La dynamique s'écrit alors :

$$\frac{\mathrm{d}V(t)}{V(t)} = \alpha \mathrm{d}t + \sigma \mathrm{d}W(t)$$

Estimation de la perte sachant le défaut

Si défaut : le créancier devient propriétaire de la firme

$$V(T) < B$$

Le payoff final s'écrit alors :

$$D(T) = B - \max\left(B - V(t)\ ,\ 0\right)$$

Processus du taux sans risque

Il est supposé constant par Merton.

Valeur de la dette

La valeur de la dette est alors donné à tout instant par

$$F = V - f$$

sachant que la valeur de marché V des actifs de la firme doit être estimée.

Spread de taux

Merton en déduit alors le spread de taux par rapport au taux sans risque :

$$R\left(T\right)-r=-\frac{1}{T}\ln\left(\Phi\left(h_2\left(d,\sigma^2T\right)\right)+\frac{1}{d}\Phi\left(h_1\left(d,\sigma^2T\right)\right)\right)$$

où :

$$\begin{cases} d=\frac{B}{V_0.e^{rT}} \\ h_1\left(d,\sigma^2T\right)=-\frac{1}{\sqrt{\sigma^2T}}\left(\frac{1}{2}\sigma^2T-\ln\left(d\right)\right) \\ h_2\left(d,\sigma^2T\right)=-\frac{1}{\sqrt{\sigma^2T}}\left(\frac{1}{2}\sigma^2T+\ln\left(d\right)\right) \end{cases}$$

Conclusion

- La prime est une fonction croissante de la volatilité
- La prime est une fonction croissante du levier

Les extentions

- Généralisation de la notion de défaut
  - prise en compte du défaut avant la maturité
    - aux dates de paiements (Geske [1977])
    - en continu sur la durée de vie
  - barrière déterministe (Black & Cox [1976])

$$K(t) = Ke^{-r(T-t)}$$

- Introduction de sauts dans le processus de valorisation des actifs (Zhou [1977])

La distance au défaut

Le point de défaut K se situe entre la dette totale et la dette à court terme

$$DD = \frac{V_A - K}{\sigma_A V_A}$$

soit le nombre d'écart-type séparant la valeur de marché des actifs du point de défaut

Caractéristiques

Cette mesure prend en compte

- Le levier des actionnaires d
- La volatilité des actifs et donc
  - l'influence du secteur d'activité
  - l'influence de la zone géographique
  - l'influence de la taille de l'entreprise

Estimation

KMV utilise la vision optionnelle du passif de la firme : (cf. volatilité implicite en trading)

Definition

La valeur de la firme est le prix d'un call européen de sous-jacent la valeur de marché des actifs et de strike la valeur comptable de la dette.

Remarque : Si le marché obligataire était plus liquide et plus efficient, nous aurions pu utiliser ce dernier pour déterminer la valeur de marché des actifs ainsi que leur volatilité.

Concrètement

Si on suppose que V_A évolue suivant une loi log-normale, on obtient le sytème

$$\begin{cases} V_{bourse} = V_A.\Phi\left(d_1\right) - e^{-rT}B.\Phi\left(d_2\right) \\ \sigma_{bourse} = \frac{V_A}{V_{bourse}}.\Phi\left(d_1\right).\sigma_A \end{cases}$$

avec

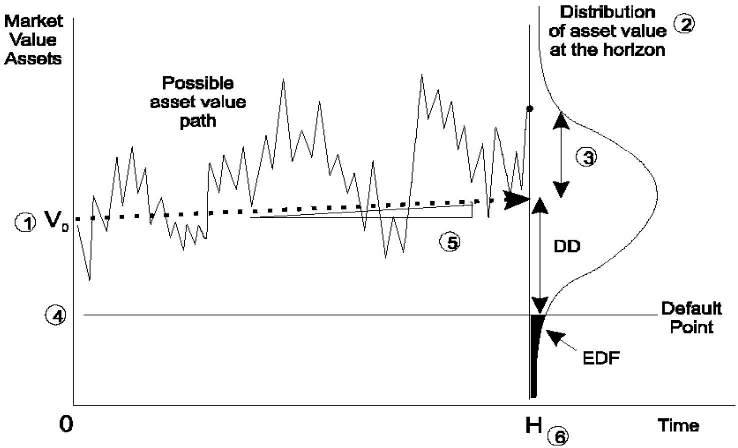
$$\begin{cases} d_1 = \frac{1}{\sigma_A.\sqrt{T}}\left(\ln\left(\frac{V_A}{B}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T\right) \\ d_2 = d_1 - \sigma_A\sqrt{T} \end{cases}$$

Expected Default Frequency

Pour le calcul des EDF, il est nécessaire de connaître

- la valeur de marché initiale des actifs
- la fonction de répartition de la valeur des actifs à l'horizon H
- la volatilité future des actifs à l'horizon H
- le niveau du point de défaut à l'horizon H
- le rendement espéré des actifs

Visuellement



Démarche

On spécifie ainsi le processus suivi par la valeur des actifs

$$\ln\left(V_A\left(T\right)\right) = \ln\left(V_A\right) + \left(\alpha + \frac{1}{2}\sigma_A^2\right) T + \sigma_A\sqrt{T}\varepsilon$$

ainsi que la distance au défaut

$$DD = \frac{\ln\left(\frac{V_A}{B}\right) + \left(\alpha + \frac{1}{2}\sigma_A^2\right) T}{\sigma_A\sqrt{T}}$$

et enfin la probabilité de défaut

$$P\left(T\right) = P\left(V_A\left(T\right)\right)$$

Limites

Trop de paramètres à connaître !

- La structure financière de l'émetteur (valeur des actifs)
- Les conditions d'émission de la dette
- La perte sachant le défaut
- Le processus suivi par le taux sans risque

Step 1 : Choose a credit model

As we said before, the Gaussian one-factor model is becoming the reference.

$$Z_i = \sqrt{\rho} X + \sqrt{1-\rho} \epsilon_i$$

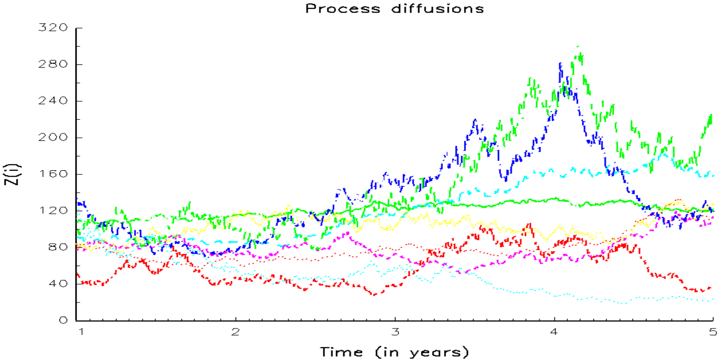
Step 2 : Construct the correlation matrix

Note that rating agencies like Moody's or S&P give methodologies that depend on factors like Corporate vs Sovereign, Industries, local vs global activities, ...

$$Z_i = \sqrt{\rho}X + \sqrt{\rho_S - \rho}X_S + \sqrt{1 - \rho_S}\epsilon_i$$

Step 3 : Diffuse your credit model

The convergence of the calculation has to be monitored carefully.



Step 4 : The correlation matrix

After a Cholesky decomposition, correl your assets for each diffusion.

Step 5 : The time of default

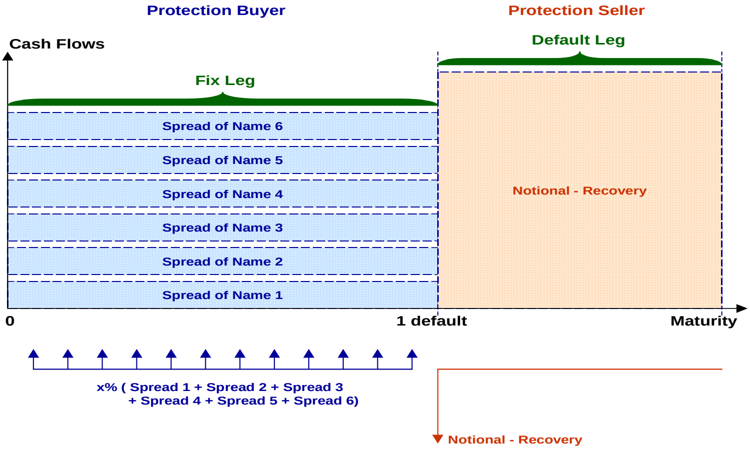
It will be obtained by comparing the level of the asset with the level of the barrier (deterministic or stochastic).

Step 6 : The break-event spread

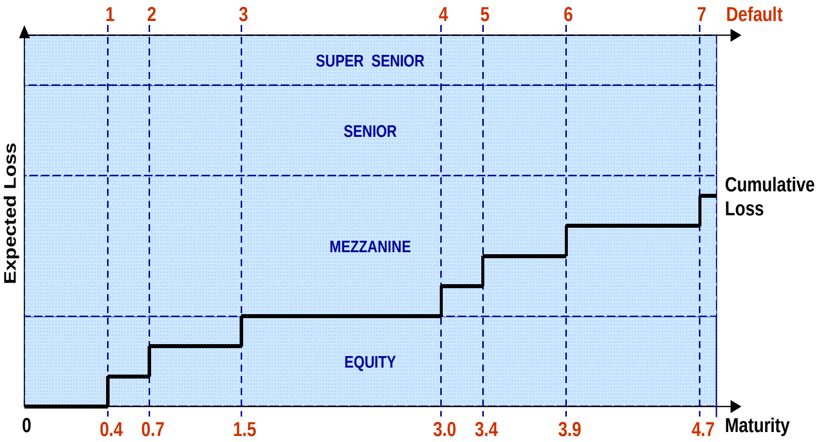
It will be obtained by matching the fix leg and the default leg.

$$\text{Spread} = \frac{\text{Floating Leg}}{\text{Fixed Leg}}$$

Visually


Protection Buyer
Protection Seller
Cash Flows
Default Leg
Fix Leg
Spread of Name 6
Spread of Name 5
Spread of Name 4
Notional - Recovery
Spread of Name 3
Spread of Name 2
Spread of Name 1
0
1 default
Maturity
x% (Spread 1 + Spread 2 + Spread 3
+ Spread 4 + Spread 5 + Spread 6)
Notional - Recovery

CDO tranching



Recently

Recently, Moody's has decided to change his methodology. The purpose is now to approach tranche ratings with a Monte Carlo framework.

In two words

The method consists in the calculation of the expected loss of a given tranche. This expected loss will then be compared to tables (calibrated by Moody's) in order to obtain the rating of the tranche.

Step 1 : The multi-sectorial factor model

Intra-sectorial correlation is supposed to be 15% and inter-sectorial correlation is 3%. The model can be written as :

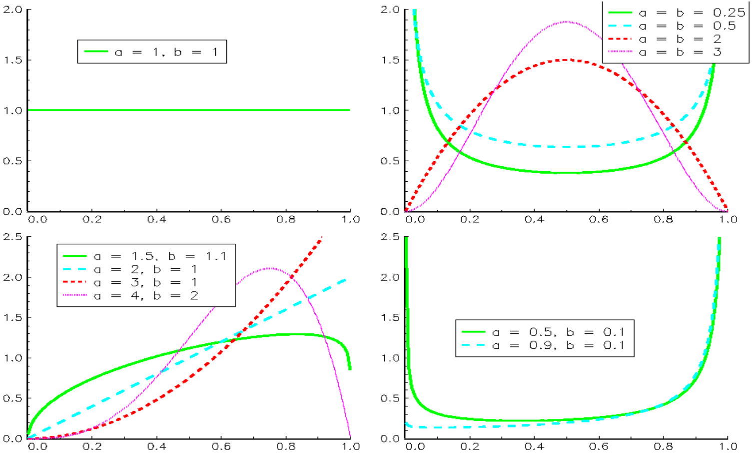
$$Z_i = \sqrt{\rho} X + \sqrt{\rho_s - \rho} X_s + \sqrt{1 - \rho_s} \varepsilon_i$$

The recovery rate of each firm is supposed to follow a Beta law which density is :

$$f(x) = \frac{x^{a-1}(1-x)^{b-1}}{\int_0^1 x^{a-1}(1-x)^{b-1} dx}$$

where $a = \frac{\mu^2(R)(1-\mu(R))}{\sigma^2(R)} - \mu(R)$ and $b = \frac{\mu(R)(1-\mu(R))^2}{\sigma^2(R)} - (1 - \mu(R))$

Some examples of Beta distribution density



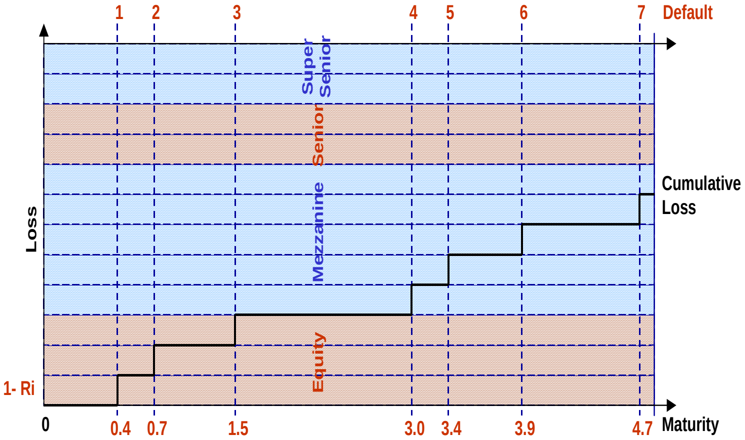
Step 2 : The Expected Loss

For each scenario $(LGD_1,...,LGD_N,Z_1,...,Z_N)$, we calculate the loss on the CDO tranche.

After averaging the loss hundred thousand simulations, we obtain the expected loss.

According to the level of the expected loss for a given tranche, Moody's gives it a rating.

Visually


1
2
3
4
5
6
7
Default
Super Senior
Senior
Mezzanine
Equity
Loss
Cumulative Loss
1- Ri
0
0.4
0.7
1.5
3.0
3.4
3.9
4.7
Maturity

Diversity Score

This concept that symbolized the equivalent number of independent firms in the CDO has disappeared.

100 firms $\Rightarrow$ DS independent firms.

$$DS \simeq \sum_{class\ c} \frac{-1 + \sqrt{1 + 8 \times n(c)}}{2}$$

WARF : Weighted Average Rating Factor

This indicator gave a first idea of the portfolio quality (the portfolio mean rating).

$$WARF = \frac{1}{N}\sum_{i=1}^{N} N_i \times RF_i$$

	stress factor	R Factor	Default proba (5Y)	Cumul Loss
Aaa	1,5	1,00	0,003%	0,00001595
Aa1	1,45	10,69	0,031%	0,0001705
Aa2	1,4	23,45	0,068%	0,000374
Aa3	1,37	48,97	0,142%	0,000781
A1	1,34	90,00	0,261%	0,0014355
A2	1,31	161,03	0,467%	0,0025685
A3	1,28	251,72	0,730%	0,004015
Baa1	1,26	379,31	1,100%	0,00605
Baa2	1,23	544,83	1,580%	0,00869
Baa3	1,2	1 051,72	3,050%	0,016775
Ba1	1,18	1 820,69	5,280%	0,02904
Ba2	1,15	2 900,00	8,410%	0,046255
Ba3	1,1	4 089,66	11,860%	0,06523
B1	1	5 558,62	16,120%	0,08866
B2	1	7 141,38	20,710%	0,113905
B3	1	9 327,59	27,050%	0,148775
Caa1	1	12 521,98	36,314%	0,19972557
Caa2	1	16 810,34	48,750%	0,268125
Caa3	1	24 076,28	69,821%	0,3840166

Mean Default Probability

The portfolio mean default probability that corresponds to this WARF was adjusted with 3 coefficients :

$$\overline{p} = \max\left(1 + \frac{Stress\ Factor(Rating_{tranche})}{Stress\ Factor(WARF)}\right)(1+\gamma^{p})p(WARF)$$

Expected Loss

The expected loss was obtained by integrating on the recovery law and the number of defaults :

$$EL = \sum_{R} \sum_{k=0}^{DS} \Pr\left(D = k\right) \Pr\left(R \,|D = k\,\right) L_{tranche}\left(R, k\right)$$

where the probability to have k defaults was given by

$$\Pr\left(D = k\right) = C_{DS}^{k} \overline{p}^{k} \left(1 - \overline{p}\right)^{DS-k}$$

Bibliographie

- www.finance-factory.fr